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Arbitrary Lagrangian Eulerian method for laser plasma simulations

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SUMMARY

The arbitrary Lagrangian Eulerian (ALE) 2D code has been developed in Cartesian and cylindrical geometries. For laser plasma simulations, the code has been extended by heat conductivity, laser absorption and QEOS equation of state. Three particular problems (originated in laser plasma experiments) for which pure Lagrangian simulation fails demonstrate the necessity of using the ALE method. Copyright © 2007 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Laser plasma, created by the interaction of laser radiation with matter, is modeled as compressible fluid by Euler equations with heat conductivity and laser absorption source term. Simulated problems typically involve large-scale corona expansion or target compression with moving boundaries. Lagrangian coordinates moving with the fluid are much more convenient for such problems than Eulerian static coordinates that are not well suited for large-scale changes of computational domain and for moving boundaries. The compressible Euler equations with heat flux and laser absorption

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terms written in the Lagrangian coordinates are

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \operatorname{div} \mathbf{u} = 0, \quad \rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} + \operatorname{grad} p = 0 \tag{1}$$

$$\rho \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + p \operatorname{div} \mathbf{u} = -\operatorname{div}(\mathbf{I}) - \operatorname{div}(\mathbf{w}) \tag{2}$$

where ρ is the density, $\mathbf{u}$ the velocity, p the pressure, ε the specific internal energy and $\mathbf{w}$ the heat flux given by $\mathbf{w} = -\kappa \operatorname{grad} T$ with T being the temperature and κ the heat conductivity. Laser absorbtion is expressed in quasistatic approximation *via* energy flux density $\mathbf{I}$ (Poynting vector). The system is closed by the equation of state $p = p(\varepsilon, \rho)$, $T = T(\varepsilon, \rho)$. The total Lagrangian time derivatives in this system include convective terms $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{u} \cdot \operatorname{grad}$. The movement of each node of the Lagrangian computational mesh is defined by an ordinary differential equation $\mathrm{d}\mathbf{x}(t)/\mathrm{d}t = \mathbf{u}$.

For many problems, e.g. those with shear flow, however, the Lagrangian moving mesh can degenerate rather soon during the simulation. By degeneration we mean that the mesh loses its regularity, some cells become non-convex or even inverted when a cell vertex crosses the edge of the same cell. When the mesh degenerates, the Lagrangian computation cannot continue as its assumptions on the mesh regularity are violated, and the simulation fails. The mesh distortion problems can be avoided by using the arbitrary Lagrangian Eulerian (ALE) method [1].

2. EMPLOYED NUMERICAL METHODS

System (1)–(2) is split into weakly hyperbolic Lagrangian hydrodynamics system (with laser absorption source term in the energy equation (2)) and parabolic heat conductivity equation.

2.1. Arbitrary Lagrangian Eulerian method

The Lagrangian hydrodynamics system is treated by the ALE method that, either after several Lagrangian time steps or when the mesh becomes distorted, rezones the mesh and remaps conservative quantities from the original Lagrangian mesh to the new rezoned smoothed mesh. After rezone and remap stage the Lagrangian computation can continue. We have developed 2D ALE code for laser plasma simulations using logically rectangular quadrilateral mesh in Cartesian or cylindrical coordinates [2]. For Lagrangian method we use compatible staggered method [3] conserving the total energy. For rezoning we employ either simple Winslow smoothing [4], or reference Jacobian method [5], or a method derived from mesh untangling [6]. For conservative interpolation in the remapping stage, we use piecewise linear limited reconstruction with either exact or swept region integration [7]. The remapping is followed by a repair [8], which conservatively redistributes conservative quantities in the new mesh, so that no new local extrema are created.

2.2. Heat conductivity and laser absorption

Heat conductivity term is treated separately by splitting from the hydrodynamics by the mimetic method [9] that works well even on bad quality Lagrangian meshes. The fully implicit discretization

$$a \frac{T^{n+1} - T^n}{\Delta t} + D\mathbf{w}^{n+1} = 0, \quad \mathbf{w}^{n+1} - GT^{n+1} = 0$$

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of the heat equation in the flux form allows to use the same time step for hyperbolic hydrodynamics and parabolic heat equation. Here, D and G are mimetic discrete operators approximating div and $-\kappa$ grad. Implicit temperature is eliminated from the difference scheme and the system for heat flux $\mathbf{w}^{n+1}$ is formed and solved numerically [9]. The norm of the heat flux $|\mathbf{w}^{n+1}|$ has to be limited by heat flux limiter derived from the ability of electrons in plasma to carry the heat energy. The classical Spitzer–Harm plasma heat conductivity [10] with corrections is used.

Laser absorption term is included in the hydrodynamical Lagrangian step as a source term in the energy equation. We either assume that laser is absorbed only on the critical surface which is the density isosurface with critical density, or we neglect laser beam refraction and for straight laser rays we solve stationary 1D Maxwell equations giving us laser intensity profiles on the rays, including both transmitted and reflected parts of laser radiation.

2.3. Cylindrical geometry

For laser plasma applications, the cylindrical r–z geometry is essential. We have generalized all employed numerical methods into the cylindrical geometry [11]. The Lagrangian step uses modification of the control volume method [3] with cell centers defined at mass centers of the cells. Both Lagrangian step and remapping phase of the ALE method involve evaluation of integrals of low degree polynomials over polygons. The cylindrical geometry compared with the Cartesian one brings only additional factor r into these integrals that are transformed by Green's theorem into integrals over the edges of the polygons and evaluated exactly. The mimetic method [9] used in the heat equation is also generalized to cylindrical geometry.

3. NUMERICAL RESULTS

On three particular laser plasma modeling problems, which we were unable to treat by pure Lagrangian simulation, we demonstrate the usefulness of the ALE method for laser plasma simulations. The problems model particular physical experiments performed at the Prague Asterix Laser System.

3.1. High-velocity impact

The first class of modeled experiments are high-velocity impact problems. A small disc is irradiated by laser beam and ablatively accelerated downwards toward the massive target. The impacting velocity of the disc flyer is in the range of 40–200 km/s. Here, we present the impact of a homogeneous 32.3-μm thick disc with radius 150 μm, density 0.656 g/cm^3 and temperature 2.1 eV. The impacting speed of the disc is 62 km/s downwards. The initial mesh with density color map is shown in Figure 1(a). The disc is the upper blue part and the massive target is the lower yellow part. The edge of the disc is smoothed. When this problem is simulated by pure Lagrangian method, the Lagrangian moving mesh becomes seriously distorted rather soon close to the disc edge as presented in Figure 1(c) at time $t = 0.16$ ns when several computational cells become non-convex and Lagrangian computation cannot continue. The mesh distortion is caused by shear flow at the disc edge and pure Lagrangian method cannot deal with this shear. On the other hand, the simulation by the ALE method (shown at the same time $t = 0.16$ ns in Figure 1(b)) continues without mesh distortion problems till the final time. Approximately spherical shock wave (located in the solid part in Figure 1(d)) propagates after the impact into the target, while disc flyer material

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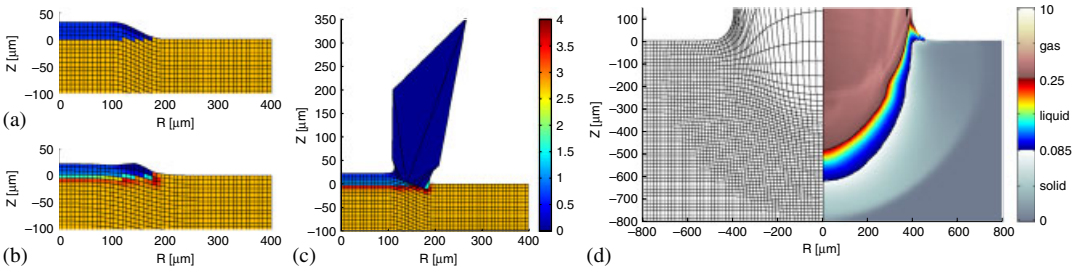

Figure 1. The high-velocity impact problem: density (in g/cm^3) profiles for: (a) initial conditions; (b) ALE; (c) pure Lagrangian results at time $t = 0.16$ ns after the impact; and (d) temperature (in eV) and the computational mesh at time $t = 70$ ns after the impact.

together with a part of target is reflected in a corona-like shape, creating a crater in the massive target. The temperature at the final time $t = 70$ ns is presented in Figure 1(d) in three different colormaps distinguishing solid, liquid and gas phase of the aluminum target. Simulated crater sizes and shapes correspond reasonably well to the experimental ones [12].

3.2. Double foil target

The double foil target of the second modeled experiment includes two parallel foils located at the distance 360 μm. The upper 0.8-μm thick foil is irradiated by a 250-ps long laser pulse with energy 78 J, focal spot radius 40 μm and wavelength 0.438 μm. The foil ablatively expands in both upward and downward directions. Rather soon laser beam burns a hole with sub-critical plasma density in the upper thin foil and starts to irradiate also the lower 2-μm thick foil that first starts to expand upwards. Between the foils the plasma corona moving downwards from the upper foil collides with the plasma corona moving upwards from the lower foil. The two plasmas touch approximately at time of laser maximum as shown in Figure 2(a) (the overcritical density, through which laser does not penetrate, is shown in dark red color). Later the collision of two plasmas produces a sphere-shaped maximum in density and pressure shown at 100 ps after the laser maximum in Figure 2(b) and (c). The temperature at the same time is presented in Figure 2(d). The temperature at the lower part is higher due to laser absorption in the lower foil. From the density and temperature plasma parameters obtained from simulations, one can compute X-ray emission spectra that can be compared with the measured spectra [13].

The problematic part of this simulation is modeling the almost vacuum state with low density and low pressure between the foils. The pure Lagrangian method is able to simulate laser interaction with single foil when the vacuum is out of the moving computational domain and influence the solution through the pressure boundary conditions. For the double foil target, we have to also cover the vacuum region between the foils by the computational mesh and at the same time the mass of the neighboring vacuum and foil cells should not differ too much. This means that foil cells neighboring the vacuum should be very small, while the vacuum cells are big. As initial mesh we use rectangular mesh aligned with coordinate axes which is geometric in radius r, uniform in vacuum region in z and geometric in foils, so that, e.g. one foil rectangular cell has r/z edge lengths ratio 10^4 and neighbors the vacuum cell with r/z ratio 0.2. When we perform the simulation with such a mesh by pure Lagrangian method, the moving mesh degenerates typically soon after the upper foil begins to expand downwards into the vacuum area and Lagrangian computation cannot

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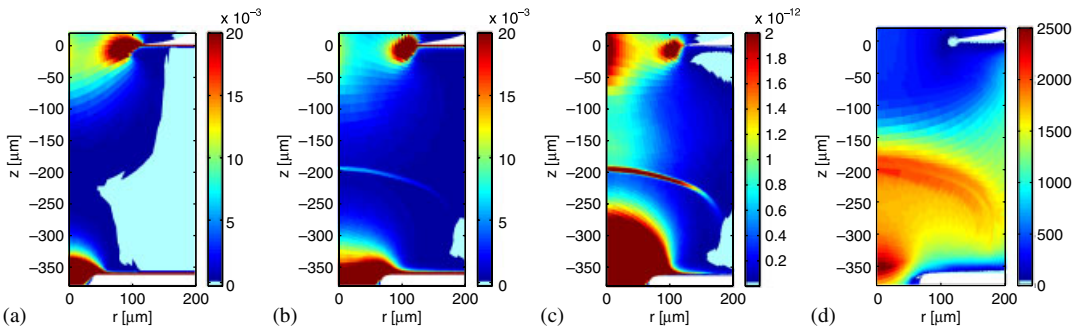

Figure 2. Double foil target, the upper foil is located at $z = 0$ and the lower foil at $z = -360\,\mu$m: (a) density at time of laser pulse maximum; (b) density at 100 ps; (c) pressure at 100 ps; and (d) temperature (in eV) at 100 ps after maximum. The bottom color of minimal values is modified to light blue showing vacuum-like areas in density and pressure and cool foils in temperature plots.

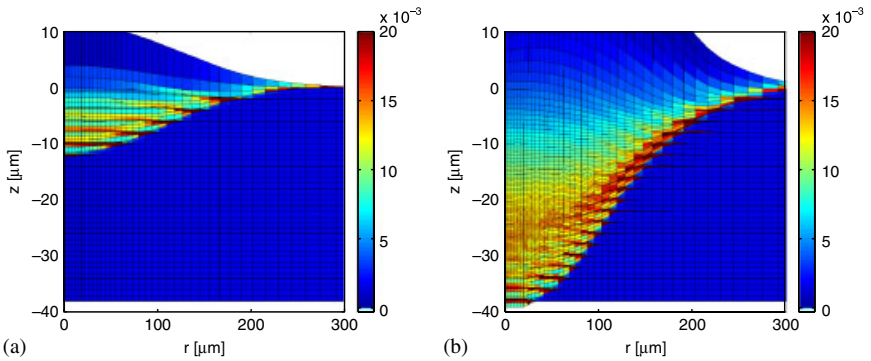

Figure 3. Density (in g/cm^3) of foam target at time (a) $t = 10$ ps and (b) $t = 30$ ps.

continue. On the other hand, the ALE method allows to simulate laser interaction with such double foil target.

3.3. Foam target

The last simulated problem is laser interaction with foam target. The foam target is modeled by the sequence of parallel foils; hence, it might be considered as a generalization of the double foil target. The laser beam gradually burns through the sequence of foils. The speed of laser (burning) penetration through such structured target is smaller than the speed in the case of modeling the foam by uniform low-density material, which might be subcritical, and closer to experimental measurements [14]. As an example we present here a foam modeled by the sequence of 0.02-μm thick dense slabs with density 1 g/cm^3 separated by 1.98-μm thick voids with density 1.43 mg/cm^3. The foam target contains 20 such slabs and is irradiated by a laser beam with focal spot radius 125 μm. The laser intensity on the z-axis grows from zero to 7.4×10^{14} W/cm^2 during the first 1 ps and then remains constant. The density of this foam target at time 10 and 30 ps computed by the ALE method is shown in Figure 3. Again pure Lagrangian method for this simulation crashes

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quite soon for reasons similar as in the case of double foil target, while the ALE method is able to perform the simulation.

4. CONCLUSION

The ALE method has been presented briefly. Three presented simulations modeling laser plasma experiments cannot be computed with pure Lagrangian method due to severe mesh distortion, while the ALE method provides reasonable results. This demonstrate the power and usefulness of the ALE method.

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